St Barnabas (VA) Primary School
Progression towards a standard method of calculation

Introduction:
The 2014 National Curriculum provides a structured and systematic approach to the teaching of calculation. The aim is for mental calculations and written procedures to be performed efficiently, fluently, accurately with understanding. Procedures and understanding are to be developed in tandem. End of key stage expectations are explicit in the programme of study.
At St Barnabas (VA) Primary School, we have a consistent approach to the teaching of written calculation methods in order to ensure continuity and progression across the school.

Age related expectations:

This calculation policy is organised according to age appropriate expectations as set out in the National Curriculum 2014, **however it is vital that pupils are taught according to the stage that they are currently working at,** being moved onto the next level as soon as they are ready, or working at a lower stage until they are secure enough to move on.

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods. It is also important for children to be confident to use mental and written strategies to explain their thinking**.** This must be a priority within calculation lessons**.** Written methods need to be viewed as tools to enable children to solve problems and record their thinking in an organised way.

Aims:
Children should be able to use an efficient method, mental or written appropriate to the given task, with understanding. By the end of year 6, children will have been taught, and be secure with, a compact standard method for each operation.

To develop efficient written calculation strategies children need:
- Secure mental methods which are developed from early years
- A solid understanding of the number system
- Practical hands on experience including a range of manipulatives
- Visual models and images including number lines and arrays
- Experience of expanded methods to develop understanding and avoid rote learning
- Secure understanding of each stage before moving onto the next.

Before carrying out a calculation, children will be encouraged to consider:

- Can I do it in my head? (using rounding, adjustment)
- The size of an approximate answer (estimation)
- Could I use jottings to keep track of the calculation?
- Do I need to use an expanded or compact written method?

Skills for written calculations

Addition and subtraction:

- Do they know all the addition and subtraction facts for all numbers to 20?
- Do they understand place value and can they partition and then re-partition numbers?
- Can they add three single digit numbers mentally?
- Can they add and subtract any pair of two digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

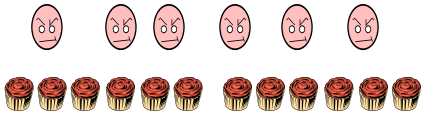
Multiplication and Division:

- Do they know the 2, 5 and 10 times tables and corresponding division facts?
- Do they know the result of multiplying by 1 and 0?
- Do they understand 0 as a place holder?
- Can they multiply two and three digit numbers by 10 and 100?
- Can they double and halve two digit numbers mentally?
- Can they use multiplication and division facts they know to derive mentally other multiplication and division facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?

These lists are not exhaustive but are a guide for the teacher as they structure the move from informal to formal methods of calculation. It is vitally important that children's mental methods of calculation continue to be practised and secured alongside their learning and use of an efficient written method for each operation.

A pathway to teaching calculation methods:

Expanded methods should be viewed as steps towards a standard method and not as methods in themselves.
Before beginning to record in a more refined written format children must have had significant practical work reinforced with appropriate manipulatives, models and images.
Teachers will guide pupils to refine their written methods of recording by modelling and asking questions such as "What is the same? What's different?"
Learning will be planned to ensure pupil are encouraged to use and apply what they have learnt to problem solving tasks.
As children move along the pathway it is vital that they **practice, reinforce, consolidate, use and apply** it to mathematical learning and NOT simply move onto the next step.
Point to note: *Teachers should refer to the programme of study for key vocabulary for each year group.*

Stage 1 (Reception)			
The Early Learning Goal for Year R is children count reliably with numbers from 1- 20 , place them in order and say which number is one more or one less than a give number. Using quantities and objects, they add and subtract two single digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.			
Addition and subtraction	**Exemplification and suggested manipulatives**	**Multiplication and division**	**Exemplification and suggested manipulatives**
Count from 1-20 Say which number is 1 more than a given number. Using quantities and objects add two single digit numbers. Addition by counting on. Say which number is 1 less than a given number. Using quantities and objects subtract two single digit numbers. Subtraction by counting back.	For one more/ less Rhymes and songs eg 5 Currant buns. Numicon, fingers, objects and toys. Adding using two groups of objects and counting them all. Objects or toys (because they can be moves into one group), then numicon. (where you are treating the numbers as objects themselves and joining the pieces together to make the total shape.) Addition by counting on – using numicon and adding one more, then counting on from the original number represented by numicon. Moving on to number cards, holding bigger number in their heads and counting on with numicon or fingers to support. Number lines used to count on/back. Double sided counters for number bonds. Subtraction by taking away and counting how many are left. Objects, skittles game, pictures on IWB which are deleted. Subtraction by counting back. Objects where the number to be subtracted has been separated from the set. Then count back whilst moving the separated pieces. Numicon using ones and again separating the number to be taken away before counting back. This is to prevent double counting, which is far too difficult at this stage. (All of this work is not formally recorded until the Summer term, however every calculation is spoken out orally eg six and four is the same as ten, ten take a way four is the same as six. When recording, the words add, subtract and equals are introduced.)	They solve problems, including doubling, halving and sharing. Combining groups of 2, 5, or 10 (Exceeding)	Doubling using fingers behind children's backs u to 5. Doubling using numicon/double sided counters Also with objects, spots on a ladybird etc. Halving using numicon and fingers as the opposite of doubling. Using object, toys, playdoh cakes and sand cakes. Pictures/Objects 12 cakes shared between 6 (not recorded)  Counting in 2's, 5's and 10's using songs, numicon, fingers, objects and toys eg. Cones placed on top of each other. Arrays – double sided counters

Key Stage 1

The principal focus of mathematics teaching in key stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with the four operations.

Stage 2 (Year 1)

Addition and subtraction

Statutory and Non statutory Guidance

- Count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number
- Count in 2's, 5's and 10's.
- Given a number, identify one more and one less
- Represent and use number bonds and related subtraction facts within 20 *(memorise and reason)*
- Add and subtract one-digit and two-digit numbers to 20, including zero
- Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ - 9.

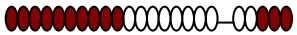
Including using quantities. Problems

Exemplification and suggested manipulatives

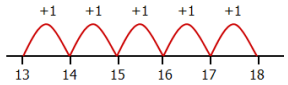
Pupils use concrete objects and pictorial representations including the number track and number line. *Numicon, balance scales, bead strings/ bead-bar unifix, coat-hanger and pegs, counting stick, songs, compare bears/dinosaurs*

Visual
(modelled using numicon, bead strings, multilink, objects, pictures, straws, number lines, songs)

13 + 5 = 18

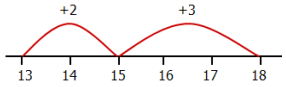


Number line, double sided counters, cubes Numicon.



Visual (efficient jumps)

13 + 5 = 18
[jumps may be in 1s]



Backward jumps for subtraction
Also finding the difference by counting up.

15 – 7 = 8
[jumps may be in 1s]

Multiplication and division

Statutory and Non statutory Guidance

- Count in multiples of twos, fives and tens
to develop their recognition of patterns in the number system (for example, odd and even numbers),
- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

Exemplification and suggested manipulatives

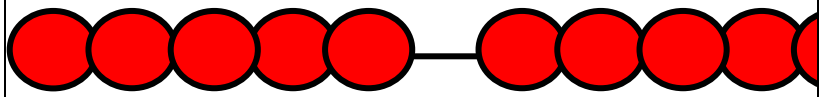
Pupils use concrete objects, pictorial representations and arrays with the support of the teacher.
e.g. double 6, double 9
½ of 10, ¼ of 8
Numicon, counting stick, bead strings, cuisinaire, cubes

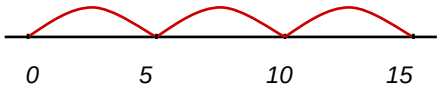
Pictures/Symbolic, numicon for arrays, bundles of straws

There are five cakes in each bag.
How many cakes are there in three bags?

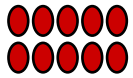
Visual (eg modelled using bead strings)

5 x 3 **or** 3 x **5** [five, three times] or [three groups of five]

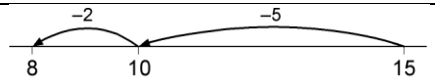
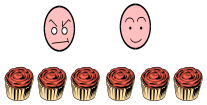
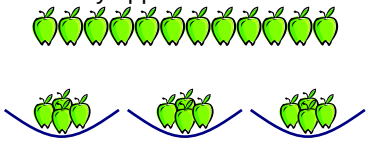
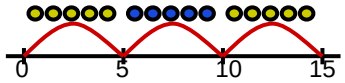
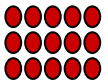


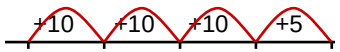
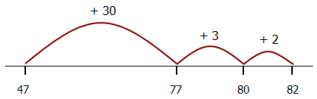
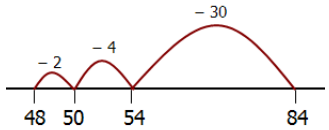
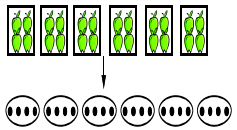
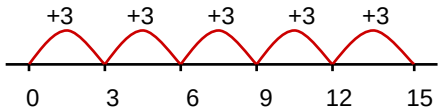
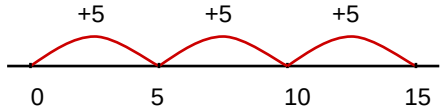
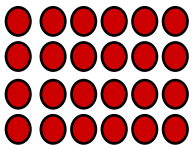


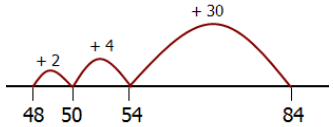
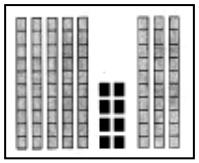
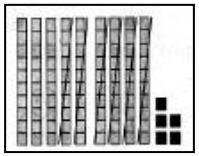
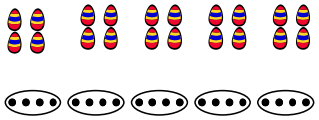
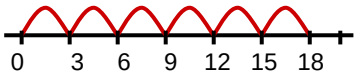
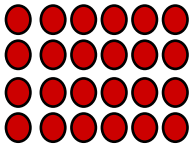
Arrays:
Numicon, counters
5 x 2 or 2 x 5



Also shown by laying numicon 2 pieces on top of a ten piece.

should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than	 Please note the jumps would take place on a fully numbered numberline. Use known facts/partitioning using straws, numicon, bead strings, dice Remember the importance of partitioning numbers in different ways to support place value in Year 2 onwards. 8 + 5 + 13 (8 + 2 = 10 10 + 3 = 13) 13 – 5 = 8 (13 – 3 = 10 10 – 2 = 8)		***Division*** Pictures/Objects Difference between sharing and grouping. 6 cakes shared between 2  6 cakes put into groups of 2  Pictures/Symbolic How many apples in each bowl if I share 12 apples between 3 bowls?  Visual (modelled using bead strings) 15 ÷ 5 = 3  Arrays 15 ÷ 5 = 3  15 is *shared* between 3

Stage 3 (Year 2)			
By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency.			
Addition and subtraction		**Multiplication and division**	
Statutory and Non statutory Guidance	**Exemplification and suggested manipulatives**	**Statutory and Non statutory Guidance**	**Exemplification and suggested manipulatives**
➢ Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 ➢ Pupils practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as using 3 + 7 = 10; 10 – 7 = 3 and 7 = 10 – 3 to calculate 30 + 70 = 100; 100 – 70 = 30 and 70 = 100 – 30. They check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, 5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5). This establishes commutativity and associativity of addition. ➢ solve problems with addition and subtraction: using concrete objects and pictorial	Pupils use concrete objects, pictorial representations and **mental strategies**. **Mental methods of addition and subtraction.** Diennes, bead strings, place value cards, number line, 100 square. Visual (efficient jumps) Using the commutative law. 35 + 47 = 82 First step   Jumps can be recorded in 10's and 1's This would be a fully numbered number line, moving to an empty number line towards the end of the year. **Taking away** 84 - 36 = 48  Jumps can be recorded in 10's and 1's	➢ Count in steps of 2, *3*, and 5 from 0, and in tens from any number, forward or backward They count in multiples of three to support their later understanding of a third. ➢ Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations. ➢ Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot. ➢ Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including	*arrays, number patterns, making connections to other aspects of the curriculum e.g. 5x table (time) 10x table (place value)* *division as sharing and grouping* *counting stick, hundred square, number line, counters, multi-line, Numicon,Cuisenaire* Pictures/Symbolic There are four apples in each box. How many apples in six boxes  Repeated addition 5 x 3 or 3 x 5   Arrays 6 x 4 or 4 x 6 

representations, including those involving numbers, quantities and measures applying their increasing knowledge of mental and written methods ➢ Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 ➢ Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: □□a two-digit number and ones □□a two-digit number and tens □□two two-digit numbers □□adding three one-digit numbers ➢ Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot ➢ Recognise and use the inverse relationship between	***(Fully numbered number line at beginning of year)*** **Counting on** 84 - 48 = 36  ***(Fully numbered number line at beginning of year)*** ***Partitioning and recombining.*** Remember the importance of partitioning numbers in different ways to support place value. 35 + 47 = 30 + 40 + 5 + 7 = 70 + 12 = 82 58 + 30 = 88  95 - 60 = 35 	problems in contexts. They begin to relate these to fraction and measures (for example, 40 ÷ 2 = 20, 20 is a half of 40). They use commutivity and inverse relations to develop multiplicative reasoning (for example, 4 x 5 = 20 and 20 ÷ 5 = 4).	***Division*** Pictures/Symbolic Four eggs fit in a box. How many boxes would you need to pack 20 eggs?  Visual (modelled using bead strings) 18 ÷ 3 = 6  Arrays Find ¼ of 24 24 ÷ 4 = 6  Partitioning 32 ÷ 2 = 16 20 ÷ 2 = 10 12 ÷ 2 = 6

addition and subtraction and use this to check calculations and solve missing number problems. ➢ Pupils should partition numbers in different ways (for example, 23 = 20 + 3 and 23 = 10 + 13) to support subtraction. They become fluent and apply their knowledge of numbers to reason with, discuss and solve problems that emphasise the value of each digit in two-digit numbers. They begin to understand zero as a place holder. ➢ Record addition and subtraction in columns to support place value and prepare for formal written methods with larger numbers.	***Recognising doubles or near doubles*** using 4 + 4 = 8 therefore 40+40 = 80 and 40 + 30 = 70 ***Complements to 10 or number bonds.*** 77 + 3 = 80 Compensating 35 + 19 = 35 + 20 -1 = 54 ***Using commutative rule- reversing additions to make them easier.*** 3 + 56 = 56 + 3 =59 ***Using associative rule- adding certain numbers together first.*** 52 + 2 + 8 = 52 + 10 = 62 ***Recognise/use inverse relationship between +/- and use to check calculations and missing number problems.*** **Informal written methods for addition and subtraction** Diennes, numicon alongside to support understanding. 47 + 35 = 82 40 + 7 30 + 5 70 + 12 *98 – 35 = 63* *90 and 8* *-30 and 5* *60 and 3 = 63*		

	Formal written methods for addition and subtraction Diennes to support Adding the units first. 47 + 76 13 110 123 *Subtraction* *9 8* *-3 5* *6 3*		

KS2

Stage 4 (Year 3)

The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.

Addition and subtraction

Statutory and Non statutory Guidance

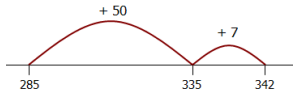
- Find 10 or 100 more or less than a given number
- Add and subtract numbers mentally, including:
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- Add and subtract numbers up to three digits, using formal written methods of columnar addition and subtraction Pupils use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent
- Estimate the answer to a calculation and

Exemplification and suggested manipulatives

Mental methods of addition and subtraction.
Using complements to ten and a hundred, number bonds, bridging through ten and a hundred, doubles and near doubles, compensating, commutative and associative rules, partitioning and recombining, and inverse operations.

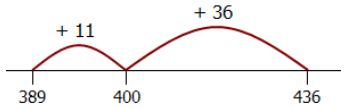
100 square, Dienne's number-line, bead string, Cuisenaire, numicon

Number line 57 + 285 = 342



Number lines as a visual image for mental calculations.

Counting on 436 - 389 = 47



Partitioning

Diennes, place value counters

57 + 285 = 342
285 + 50 = 335

Multiplication and division

Statutory and Non statutory Guidance

- Count from 0 in multiples of 4, 8, 50 and 100
- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables. Through doubling, they connect the 2, 4 and 8 multiplication tables.
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods. Using commutativity (e.g. 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and multiplication and division facts (e.g. using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
- Solve problems, including missing number problems, involving multiplication

Exemplification and suggested manipulatives

e.g. 8 x 6, 3 x 11, 15 x 4,
commutative law, arrays, counting stick, multiplication square , Numicon, counters

Informal recording might be:

$$\begin{array}{ccccc} 43 & & & & \\ 40 & + & 3 & & \\ \downarrow & & \downarrow & & \times 6 \\ 240 & + & 18 & = & 258 \end{array}$$

We would record this as:

40x6=

3x6=

Also record mental multiplication using partitioning:

$$14 \times 3 = (10+4) \times 3$$
$$= (10 \times 3) + (4 \times 3) = 30 + 12 = 42$$

$$43 \times 6 = (40+3) \times 6$$
$$= (40 \times 6) + (3 \times 6) = 240 + 18 = 258$$

Note: These methods are based on the distributive law. Children should be introduced to the principle of this law in Year 3 and 4, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7:

Shown using arrays of counters.

○○○○○○○ ○○○○○...○○
○○○○○○○ ○○○○○...○○
○○○○○○○ ○○○○○...○○

$$7 \times 3 = (5+2) \times 3 = (5 \times 3) + (2 \times 3) = 15 + 6 = 21$$

X	30	6
4	120	24

Grid Method

36 x 4 = 144

use inverse operations to check answers

They use larger numbers to at least 1000, applying partitioning related to place value using varied and increasingly complex problems, building on work in year 2 (for example, 146 = 100 + 40 and 6, 146=130 + 16).

- Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.
- Count up and down in tenths recognise that tenths arise from dividing an object into10 equal parts and in dividing one-digit numbers or quantities by 10
- Add and subtract fractions with the same denominator within one whole (e.g. $^{5}/_{7} + {}^{1}/_{7} = {}^{6}/_{7}$)
- Solve problems which involve fractions.

335 + 7 = 342

Taking away by partitioning

326 - 178 = 148
326 - 100 = 226
226 - 70 = 156
156 - 6 = 150
150 - 2 = 148

Then partitioning into columns as in Year 2

Informal written methods for addition and subtraction

Expanded
Vertical
Modelled
Using e.g.
Diennes
Place value
Counters

$$\begin{array}{r} 374 \\ +\ 248 \\ \hline 12 \\ 110 \\ 500 \\ \hline 622 \end{array}$$

NB: With any column calculation always start with the units.

Decomposition
Using e.g.
Diennes
Place value
Counters – double sided

723 - 458 = 265

700	20	3
400	50	8

600	110	13
400	50	8
200	60	5

and division including positive integer scaling problems and correspondence problems in which n objects are connected to m objects. (For example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

- recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators (1/3, 2/4, ¾)

Pupils connect tenths to place value, decimal measures and to division by 10.

Cuisenaire alongside for understanding.

36 x 4 = 144

30 x 4 = 120

6 x 4 = 24

36 x 4 = 144

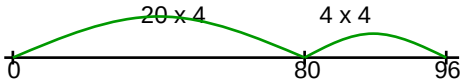
$$\begin{array}{rr} & 36 \\ & \times\ 4 \\ (6 \times 4) & 24 \\ (30 \times 4) & 120 \\ \hline & 144 \end{array}$$

36 x 4 = 144

$$\begin{array}{r} 36 \\ \times\ 4 \\ \hline 144 \\ {\scriptstyle 2} \end{array}$$

Division

96 ÷ 4 = 24



Multiples of the divisor)

85 ÷ 5 = 17

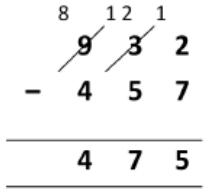
10 x 5 = 50

7 x 5 = 35

$$\begin{array}{rl} 51 & \\ 30 & (3 \times \mathbf{10}) \\ \hline 21 & \\ 21 & (3 \times \mathbf{7}) \\ \hline 0 & \end{array}$$

51 ÷ 3 =17

	We would use the subtraction sign at the side and write "and" or leave a gap between the numbers. **Formal written methods for addition and subtraction** Compact vertical Modelled Using e.g. Diennes Place value Counters $\begin{array}{r} 374 \\ +\ 248 \\ \hline 622 \\ \hline 1\ 1 \end{array}$ 874 - 523 = 351 (no decomposition $\begin{array}{r} 8\ 7\ 4 \\ -\ 5\ 2\ 3 \\ \hline 3\ 5\ 1 \end{array}$ **Decomposition** Using e.g. Diennes Place value Counters 932 - 457 = 475		51 ÷ 3 =17 $3\overline{)51}$ (17)

	 Remember to always estimate answers and use inverse to check. **Fractions addition and subtraction** Number line, Fraction walls, fraction stack, fraction action. Paper cut up, cuisinaire- longer block with 2 blocks that make up the same amount next to it. Tenths/ hundredths- use bead strings and diennes. Remember the importance of partitioning numbers in different ways to support place value.		

Stage 5 (Year 4)			
By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.			
Addition and subtraction		**Multiplication and division**	
Statutory and Non statutory Guidance	**Exemplification and suggested manipulatives**	**Statutory and Non statutory Guidance**	**Exemplification and suggested manipulatives**
➢ Find 1000 more or less than a given number ➢ Count backwards through zero to include negative numbers They practise counting using simple fractions and decimal fractions, both forwards and backwards ➢ Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate ➢ Estimate and use inverse operations to check answers to a calculation ➢ Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why. ➢ Add and subtract	**Mental methods of addition and subtraction.** Using complements to ten and a hundred, number bonds, bridging through ten, a hundred and a thousand, doubles and near doubles, compensating, commutative and associative rules, partitioning and recombining, and inverse operations. *Numicon, number line,100 square, Dienne's bead strings, Cuisenaire* *place value arrow cards, place value counters, counting stick.* *Number lines as a visual image for mental calculations.* *Subtraction by counting on with a number line.* Estimate and use inverse operations to check answers to a calculation. **Informal written methods for addition and subtraction** Compact vertical Modelled Using e.g. Diennes Place value counters until proficient. 789 + 642 = 1431 789 + 642 11 120 1300 1431 Decomposition: 1374 - 968 = 406 Place value counters alongside until proficient.	➢ Count in multiples of 6, 7, 9, 25 and 1000 Using a variety of representations, including measures, pupils become fluent in the order and place value of numbers beyond 1000, including counting in tens and hundreds, and maintaining fluency in other multiples through varied and frequent practice. ➢ Recall multiplication and division facts for multiplication tables up to 12 × 12 ➢ Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers Pupils practise mental methods and extend this to three-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6) ➢ Recognise and use factor pairs and commutativity in mental calculations Pupils write statements	*commutative, associative and distributive laws, array, scaling, factors* *Numicon, Cuisenaire, Place value cards/slider* Informal recording might be: 43 40 + 3 ×6 240 + 18 = 258 We would record this as: 40x6= 3x6= Also record mental multiplication using partitioning: $14 \times 3 = (10 + 4) \times 3$ $= (10 \times 3) + (4 \times 3) = 30 + 12 = 42$ $43 \times 6 = (40 + 3) \times 6$ $= (40 \times 6) + (3 \times 6) = 240 + 18 = 258$ Note: These methods are based on the distributive law. Children should be introduced to the principle of this law in Years 3 and 4, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7: Shown using arrays of counters ○○○○○○○ ○○○○○...○○ ○○○○○○○ ○○○○○...○○ ○○○○○○○ ○○○○○...○○ $7 \times 3 = (5 + 2) \times 3 = (5 \times 3) + (2 \times 3) = 15 + 6 = 21$ 43 x 6 = 258 (estimate: 40 x 6 = 240) 40 x 6 = 240 3 x 6 = 18

fractions with the same denominator	1300 and 60 and 14 − 900 and 60 and 8 400 and 0 and 6 **Formal written methods for addition and subtraction** **(Use inverse to check.)** 789 + 642 = 1431 Place value counters alongside until proficient. 7 8 9 + 6 4 2 1 4 3 1 1 1 **Decomposition** 932 - 457 = 475 8 12 1 9 3 2 − 4 5 7 4 7 5 **Use inverse to check.** **Fractions addition and subtraction** Number line, Paper cut up, cuisinaire- longer block with 2 blocks that make up the same amount next to it. 1/5 + 2/5 = 3/5 1/5 | 1/5 | 1/5	about the equality of expressions (for example, use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, 2 x 6 x 5 = 10 x 6 = 60. ➢ Multiply two-digit and three-digit numbers by a one-digit number using formal written layout ➢ Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects. ➢ Count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten. ➢ Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number Pupils use factors and	342 x 7 = 2394 x 300 40 2 7 2100 280 14 Support using cuisennaire. 43 x 6 = 258 43 x 6 18 (3 x 6) 240 (40 x 6) 258 24 x 6 = 144 2 4 × 6 1 4 4 2 237 × 4 (estimate: 250 × 4 = 1000) 237 × 4 28 120 800 948

	4/5 – 1/5 = 3/5 1/5 | 1/5 | 1/5 | 1/5 Cut off 1/5 Tenths/ hundredths- use bead strings and diennes. Remember the importance of partitioning numbers in different ways to support place value.	multiples to recognise equivalent fractions and simplify where appropriate (for example, 6/9 = 2/3 or ¼ =2/8) ➢ Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as units, tenths and hundredths	Support using place value counters. 342 x 7 = 2394 3 4 2 × 7 2 3 9 4 2 1 **Division** Cuisinaire and the numicon number line. Number lines, bead strings, numicon. Multiples of the divisor 98 ÷ 7 = 14 10 x 7 = 70 4 x 7 = 28 Also record mental division using partitioning: 64 ÷ 4 = (40 + 24) ÷4 = (40 ÷ 4) + (24 ÷ 4) = 10 + 6 = 16 87 ÷ 3 = (60 + 27) ÷ 3 = (60 ÷ 3) + (27 ÷ 3) = 20 + 9 = 29 Remainders after division can be recorded similarly. 96 ÷ 7 = (70 + 26) ÷ 7 = (70 ÷ 7) + (26 ÷ 7) = 10 + 3 R5 = 13R5 1 4 7 | 9 8 2 98 ÷ 7 = 14

			252 ÷ 7 = 36 30 x 7 = 210 6 x 7 = 42 252 ÷ 7 = 36 252 210 (7 x **30**) 42 42 (7 x **6**) 0 252 ÷ 7 = 36 36 7 | 252

Stage 6 (Year 5)

At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems.

Addition and subtraction

Statutory and Non statutory Guidance	Exemplification and suggested manipulatives
Addition and subtraction	
➢ Pupils extend counting from year 4, using decimals and fractions including bridging zero, for example on a number line. ➢ Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) ➢ Add and subtract numbers mentally with increasingly large numbers. They mentally add and subtract tenths, and one-digit whole numbers and tenths. ➢ Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy ➢ Solve addition and subtraction multi-	**Mental methods of addition and subtraction.** Using complements to ten and a hundred, number bonds, bridging through ten, a hundred and a thousand, doubles and near doubles, compensating, commutative and associative rules, partitioning and recombining, and inverse operations. *Numicon, number line,100 square, Dienne's bead strings, Cuisenair, bundles of straws* *place value arrow cards, place value counters, counting stick. Place value chart* *Number lines as a visual image for mental calculations.* *Subtraction on a number line as counting on.* Estimate and use inverse operations to check answers to a calculation. **Informal written methods for addition and subtraction** **Expanded** vertical Modelled using Place value counters until proficient. 23.70 + 48.56 0.06 1.20 11.00 60.00 72.26

Multiplication and division

Statutory and Non statutory Guidance	Exemplification and suggested manipulatives
Multiplication and division	
➢ Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 ➢ Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers. Pupils use and explain the equals sign to indicate equivalence, including in missing number problems (for example, 13 + 24 = 12 + 25; 33 = 5 x □). ➢ Multiply and divide numbers mentally drawing upon known facts They apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations. ➢ Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret	*Cuisenaire, Numicon, ITP, bundles of straws* 47 x 36 = 1692 (estimate 50 x 40 = 2000) × | 40 | 7 | 30 | 1200 | 210 | 1410 6 | 240 | 42 | 282 1692 27 x 34 = 918 (estimate 30 x 30 = 900) **27** **x 34** **28** (7 x 4) **80** (20 x 4) **210** (7 x 30) **600** (20 x 30) **918** 2741 x 6 = 16446 (estimate 3000 x 6 = 18000 2 7 4 1 × 6 1 6 4 4 6 4 2 2 2 4 × 1 6 2 4 0 1 4 4 3 8 4 24 x 16 = 384 (estimate 25 x 15 = 375)

step problems in contexts, deciding which operations and methods to use and why.

- Solve problems involving addition, subtraction, (multiplication and division) and a combination of these, including understanding the meaning of the equals sign

- Add and subtract fractions with the same denominator and denominators that are multiples of the same number

Pupils practise adding and subtracting fractions to become fluent through a variety of increasingly complex problems. They extend their understanding of adding and subtracting fractions to calculations that exceed 1 as a mixed number.

Taking away
(Partitioning)

72.5 – 45.7

72.5 – 40 = 32.5
32.5 – 5 = 27.5
27.5 – 0.7 = 26.8

Formal written methods for addition and subtraction
Compact vertical
Modelled using
Place value counters until proficient.
'Carrying'

```
  23.70
+ 48.56
  72.26
```

1 1 (cross off when used)

Decomposition: exchanging

72.5 - 45.7 = 26.8

```
  6 11
  7 2.15
-   4 5.7
  ------
  2 6.8
```

Fractions addition and subtraction
Number line, Paper cut up, cuisinaire- longer block with 2 blocks that make up the same amount next to it.

3/12 + 1/8 =
With squared diagram
Start with the largest number 12. 8 is not a multiple of 12 so double 12 and 8 is a multiple of 24. Make rectangle split into 24 squares. Work out how many squares 1/12 and 1/8 are compared with 1/24, by using parallel strips divided into 1/8ths and 1/12ths.

remainders appropriately for the context.
Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, 98 ÷ 4 = 98/4 = 24 r 2 = 241/2 = 24.5 ≈ 25).

- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Distributivity can be expressed as $a(b + c) = ab + ac$.

- They understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example,

4 x 35 = 2 x 2 x 35;
3 x 270 = 3 x 3 x 9 x 10 = 9^2 x 10

- Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes, scaling and simple fractions including understanding the meaning of the equals sign.

124 x 26 = 3224 [see Y6]

```
    1 2
    1 2 4
  x   2 6
  -------
  2 4 8 0
    7 4 4
  -------
  3 2 2 4
  1 1
```

(carry on bottom)

Division: *place value counters*
Mental division using partitioning:
(as in Year 4)
64 ÷ 4 = (40 + 24) ÷4
= (40 ÷ 4) + (24 ÷ 4)
= 10 + 6 = 16
87 ÷ 3 = (60 + 27) ÷ 3
= (60 ÷ 3) + (27 ÷ 3)
= 20 + 9 = 29
Remainders after division can be recorded similarly.
96 ÷ 7 = (70 + 26) ÷ 7
= (70 ÷ 7) + (26 ÷ 7)
= 10 + 3 R5 = 13R5

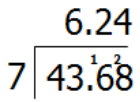
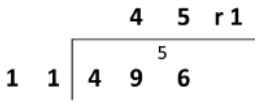
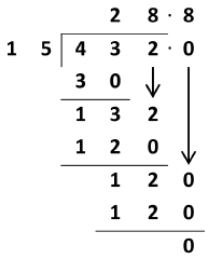
346 ÷ 8 = 43 r2 (estimate >40, <50)

	Colour correct number of squares for 3/12 and 1/8 = 9/24 = 3/8 3/12 + 1/8 = Formal method 1. Find the common denominator. 2. Express each fraction as its equivalent fraction with the common denominator. 3. Add numerators and place over common denominator. 4. Simplify. Tenths/ hundredths- use bead strings and diennes. Remember the importance of partitioning numbers in different ways to support place value.	Pupils use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres. ➢ Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams. Pupils connect equivalent fractions > 1 that simplify to integers with division and other fractions > 1 to division with remainders, using the number line and other models, and hence move from these to improper and mixed fractions. Pupils connect multiplication by a fraction to using fractions as operators (fractions of), and to division, building on work from previous years. This relates to scaling by simple fractions, including fractions > 1. ➢ Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents ➢ Solve problems involving number up to three	346 ÷ 8 (estimate: 400 ÷ 8 = 50) $8\overline{)346}$ $-\underline{320}$ (8×40) 26 $-\underline{24}$ (8×3) 2 432 ÷ 5 = 86 r2 (estimate: 400 ÷ 5 = 80) $5\overline{)432}$ = 86 r2 (3) 8520 ÷ 6 = 1420 $6\overline{)8520}$ = 1420 (2, 1) NB: Remainders need to be shown as decimals, fractions and remainders, so children see the link between the 3. **Fractions multiplication** 2/3 x 2 = 1/2 Use cuisinaire and lay out 2 of the 3 rods then another 2. 4/3 = 1 1/3 Fraction stacks, fraction action Or use diagrams- 1/3 | 1/3 | 1/3

		decimal places ➢ Solve problems which require knowing percentage and decimal equivalents of $^{1}/_{2}$, $^{1}/_{4}$, $^{1}/_{5}$, $^{2}/_{5}$, $^{4}/_{5}$ and those with a denominator of a multiple of 10 or 25. ➢ Pupils should make connections between percentages, fractions and decimals (for example, 100% represents a whole quantity and 1% is 1/100, 50% is 50/100, 25% is 25/100) and relate this to finding 'fractions of'.	<table><tr><td>1/3</td><td></td><td></td></tr></table> 4/3 = 1 1/3 Fraction walls, fraction strips, fraction stacks, fraction action, place value decimal cards

Stage 7 (Year 6)			
By the end of year 6, pupils should be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.			
Addition and subtraction		**Multiplication and division**	
Statutory and Non statutory Guidance	**Exemplification and suggested manipulatives**	**Statutory and Non statutory Guidance**	**Exemplification and suggested manipulatives**
➢ Solve number and practical problems involving numbers up to 10 000 000, using negative numbers in context and calculating intervals across zero. Pupils practise addition, subtraction, for larger numbers, using the formal written methods of columnar addition and subtraction. ➢ ➢ perform mental calculations, including with mixed operations and large numbers. ➢ solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why ➢ add and subtract fractions with different denominators and mixed numbers,	**Mental methods of addition and subtraction.** Using complements to ten and a hundred, number bonds, bridging through ten, a hundred and a thousand, doubles and near doubles, compensating, commutative and associative rules, partitioning and recombining, and inverse operations. *Numicon, number line,100 square, Dienne's bead strings, Cuisenaire* *place value arrow cards, place value counters, counting stick.* *Number lines as a visual image for mental calculations.* *Subtraction on a number line using counting on method.* Estimate and use inverse operations to check answers to a calculation. **Informal written methods for addition and subtraction** **Expanded** vertical Modelled using Place value counters until proficient. **Expanded** vertical 3.243 + 18.070 = **21.313** 3.243 + 18.070 0.003 0.110 0.200 21.000 21.313 **Formal written methods for addition and**	➢ Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency. ➢ multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication ➢ divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context ➢ divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context ➢ solve problems involving (addition, subtraction,) multiplication and division	*Cuisenaire, Numicon, place value counters* *Numbers up to 4 digits.* 256 x 18 = 4608 (estimate 250 x 20 = 5000) 256 × 18 2560 2048 4608 1 124 x 26 = 3224 1 2 1 2 4 × 2 6 7 4 4 2 4 8 0 3 2 2 4 1 1 NB See Y5 method 4.7 x 8 = 37.6

using the concept of equivalent fractions They should start with fractions where the denominator of one fraction is a multiple of the other (for example, 1/2 + 1/8 = 5/8) and progress to varied and increasingly complex problems.	**subtraction** Compact vertical Modelled using Place value counters until proficient. **Compact** vertical 3.243 + 18.070 21.313 1 1 **Subtraction by decomposition as in Year 5.** **Fractions addition and subtraction** Number line, Paper cut up, cuisinaire- longer block with 2 blocks that make up the same amount next to it. 3/12 + 1/8 = With squared diagram Start with the largest number 12. 8 is not a multiple of 12 so double 12 and 8 is a multiple of 24. Make rectangle split into 24 squares. Work out how many squares 1/12 and 1/8 are compared with 1/24, by using parallel strips divided into 1/8ths and 1/12ths. Colour correct number of squares for 3/12 and 1/8 = 9/24 = 3/8 3/12 + 1/8 = Formal method 1. Find the common denominator. 2. Express each fraction as its equivalent fraction with the common denominator. 3. Add numerators and place over common denominator. 4. Simplify. Tenths/ hundredths- use bead strings and diennes. Remember the importance of partitioning numbers in different ways to support place value.	➢ multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. $^1/_4 \times {}^1/_2 = {}^1/_8$) Pupils should use a variety of images to support their understanding of multiplication with fractions. This follows earlier work about fractions as operators (fractions of), as numbers, and as equal parts of objects, for example as parts of a rectangle. ➢ ➢ divide proper fractions by whole numbers (e.g. $^1/_3 \div 2 = {}^1/_6$) They practise calculations with simple fractions and decimal fraction equivalents to aid fluency, including listing equivalent fractions to identify fractions with common denominators. ➢ ➢ associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 3/8] pupils use their understanding of the relationship between unit fractions and division to work backwards by multiplying a quantity that represents a unit fraction	(estimate 5 x 8 = 40) 4.7 × 8 37.6 6 [Or 47 x 8, then divide the solution by 10.] 5.65 x 9 = 50.85 (estimate 6 x 9 = 54) × | 5 | 0.6 | 0.05 | 9 | 45 | 5.4 | 0.45 | 50.85 [Or compute 565 x 9, then divide the solution by 100.] **Division** 43.4 ÷ 7 = 6.2 (estimate 42 ÷ 7 = 6) 6 x 7 = 42 0.2 x 7 = 1.4 25.6 ÷ 7 = 3.2 (estimate >3, <4) **25.6 ÷ 8** (estimate: 24 ÷ 8 = 3) 8)25.6 −24.0 (8 × 3.0) 1.6 −1.6 (8 × 0.2) 0

		to find the whole quantity (e.g. if $^1/_4$ of a length is 36cm, then the whole length is 36 × 4 = 144cm). ➢	
		➢ identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places	43.68 ÷ 7 = 6.24 (estimate: 42 ÷ 7 = 6) [Or compute 4368 ÷ 7, then divide the solution by 100.]
		➢ multiply one-digit numbers with up to two decimal places by whole numbers	
		➢ use written division methods in cases where the answer has up to two decimal places	496 ÷ 11 (estimate 500 ÷ 10 = 50)
		➢ Pupils can explore and make conjectures about converting a simple fraction to a decimal fraction (for example, 3 ÷ 8 = 0.375). For simple fractions with recurring decimal equivalents, pupils learn about rounding the decimal to three decimal places, or other appropriate approximations depending on the context. Pupils multiply and divide numbers with up to two decimal places by one-digit and two-digit whole numbers. Pupils multiply decimals by whole numbers, starting with the	 Answer: $45\frac{1}{11}$  432÷ 15 = 28.8

		simplest cases, such as 0.4 × 2 = 0.8, and in practical contexts, such as measures and money. ➢ Pupils are introduced to the division of decimal numbers by one-digit whole number, initially, in practical contexts involving measures and money. They recognise division calculations as the inverse of multiplication.	NB: Remainders need to be shown as decimals, fractions and remainders, so children see the link between the 3. **Fractions multiplication and division** **2/3 x ¾ = 2/4 = ½** **Use cuisinaire and place 2 of the 3 rods next to 3 of the 4 rods. There will be 6 out of the 12, which is 6/12 = ½.** **Using diagrams-** **o** **¼** **o o** **o** **¾** **o o** **x** **1/3 of ¾** **x o** **x** **2/3 of ¾ = 2/4 = 1/2** **1/3 ÷ 2 = 1/6** **Use of fraction walls and cutting up paper.**